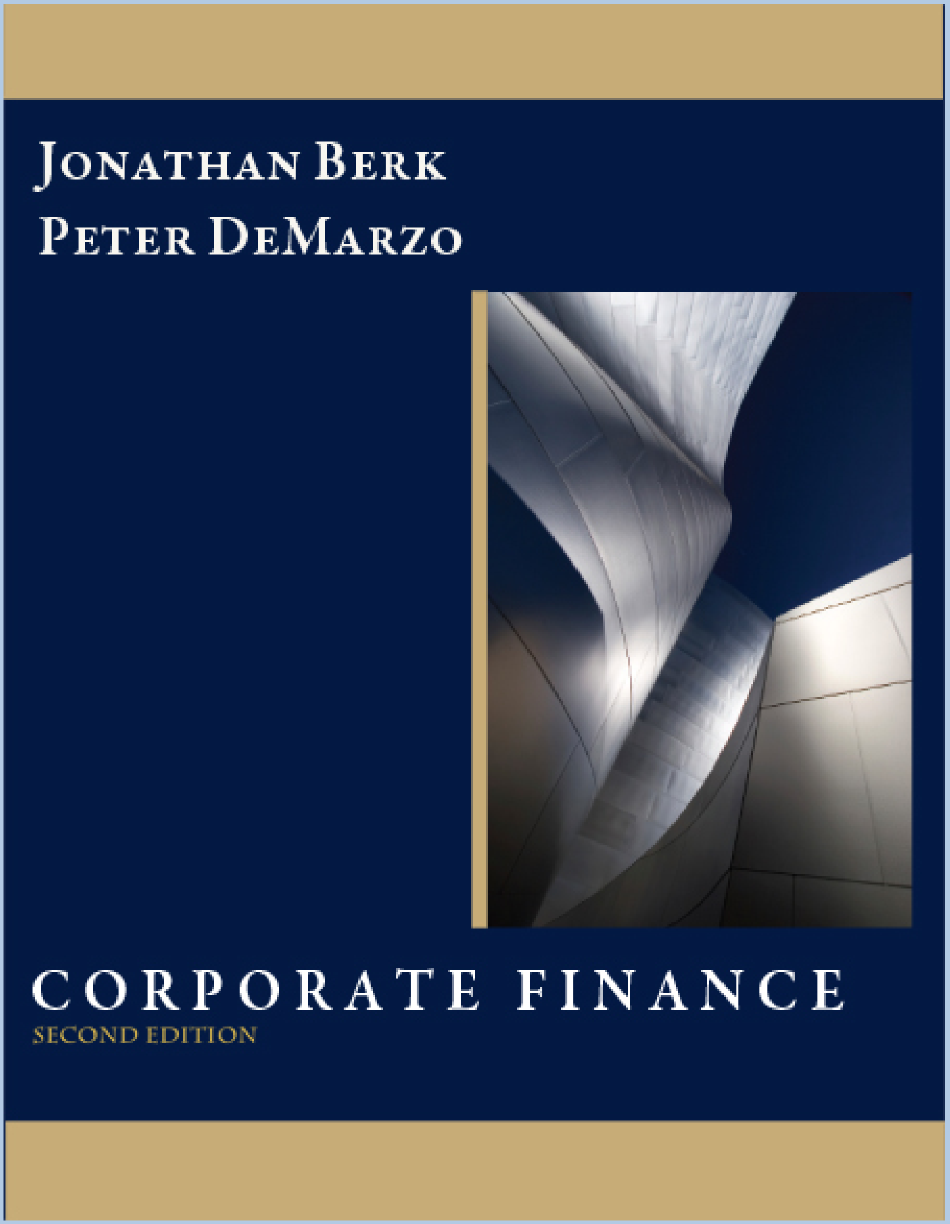
Chapter 8

Valuing Bonds



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Chapter Outline

Learning Objectives

1. Identify the cash flows for both coupon bonds and zero-coupon bonds, and calculate the value for each type of bond.
2. Calculate the yield to maturity for both coupon and zero-coupon bonds, and interpret its meaning for each.
3. Given coupon rate and yield to maturity, determine whether a coupon bond will sell at a premium or a discount; describe the time path the bond's price will follow as it approaches maturity, assuming prevailing interest rates remain the same over the life of the bond.

Learning Objectives

4. Illustrate the change in bond price that will occur as a result of changes in interest rates; differentiate between the effect of such a change on long-term versus short-term bonds.
5. Discuss the effect of coupon rate to the sensitivity of a bond price to changes in interest rates.
6. Define duration, and discuss its use by finance practitioners.
7. Calculate the price of a coupon bond using the Law of One Price and a series of zero-coupon bonds.

Learning Objectives

8. Discuss the relation between a corporate bond's expected return and the yield to maturity; define default risk and explain how these rates incorporate default risk.
9. Assess the creditworthiness of a corporate bond using its bond rating; define default risk.

8.1 Bond Cash Flows, Prices, and Yields

- Bond Terminology
  - Bond Certificate
    - States the terms of the bond
  - Maturity Date
    - Final repayment date
  - Term
    - The time remaining until the repayment date
  - Coupon
    - Promised interest payments

8.1 Bond Cash Flows, Prices, and Yields (cont'd)

- Bond Terminology
  - Face Value
    - Notional amount used to compute the interest payments
  - Coupon Rate
    - Determines the amount of each coupon payment, expressed as an APR
  - Coupon Payment

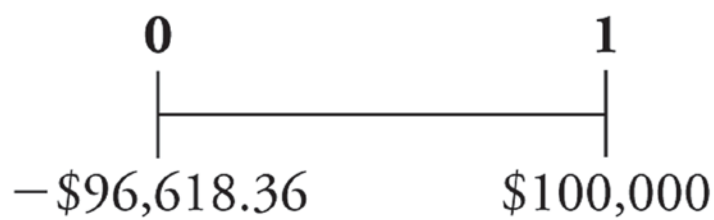
$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

Zero-Coupon Bonds

- Zero-Coupon Bond
  - Does not make coupon payments
  - Always sells at a **discount** (a price lower than face value), so they are also called **pure discount bonds**
  - **Treasury Bills** are U.S. government zero-coupon bonds with a maturity of up to one year.

Zero-Coupon Bonds (cont'd)

- Suppose that a one-year, risk-free, zero-coupon bond with a $100,000 face value has an initial price of $96,618.36. The cash flows would be:



  – Although the bond pays no “interest,” your compensation is the difference between the initial price and the face value.

Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - *The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.*
    - *Price of a Zero-Coupon bond*

$$P = \frac{FV}{(1 + YTM_n)^n}$$

Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - For the one-year zero coupon bond:

$$96{,}618.36 = \frac{100{,}000}{(1 + YTM_1)}$$

$$1 + YTM_1 = \frac{100{,}000}{96{,}618.36} = 1.035$$

- Thus, the YTM is 3.5%.

Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - Yield to Maturity of an *n*-Year Zero-Coupon Bond

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1$$

Textbook Example 8.1

Yields for Different Maturities

Problem

Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

Maturity	1 Year	2 Years	3 Years	4 Years
Price	$96.62	$92.45	$87.63	$83.06

Textbook Example 8.1 (cont'd)

Solution

Using Eq. 8.3, we have

$$YTM_1 = (100/96.62) - 1 = 3.50\%$$
$$YTM_2 = (100/92.45)^{1/2} - 1 = 4.00\%$$
$$YTM_3 = (100/87.63)^{1/3} - 1 = 4.50\%$$
$$YTM_4 = (100/83.06)^{1/4} - 1 = 4.75\%$$

Alternative Example 8.1

- **Problem**
  - Suppose that the following zero-coupon bonds are selling at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

Maturity	1 year	2 years	3 years	4 years
Price	$98.04	$95.18	$91.51	$87.14

Alternative Example 8.1 (cont'd)

- **Solution:**

$$\text{YTM} = (100 / 98.04) - 1 = 0.02 = 2\%$$

$$\text{YTM} = (100 / 95.18)^{1/2} - 1 = 0.025 = 2.5\%$$

$$\text{YTM} = (100 / 91.51)^{1/3} - 1 = 0.03 = 3\%$$

$$\text{YTM} = (100 / 87.14)^{1/4} - 1 = 0.035 = 3.5\%$$

Zero-Coupon Bonds (cont'd)

- Risk-Free Interest Rates
  - A default-free zero-coupon bond that matures on date *n* provides a risk-free return over the same period. Thus, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.
  - Risk-Free Interest Rate with Maturity *n*

$$r_n = YTM_n$$

Zero-Coupon Bonds (cont'd)

- Risk-Free Interest Rates
  - Spot Interest Rate
    - Another term for a default-free, zero-coupon yield
  - Zero-Coupon Yield Curve
    - A plot of the yield of risk-free zero-coupon bonds as a function of the bond's maturity date

Coupon Bonds

- Coupon Bonds
  - Pay face value at maturity
  - Pay regular coupon interest payments
- Treasury Notes
  - U.S. Treasury coupon security with original maturities of 1–10 years
- Treasury Bonds
  - U.S. Treasury coupon security with original maturities over 10 years

Textbook Example 8.2

The Cash Flows of a Coupon Bond

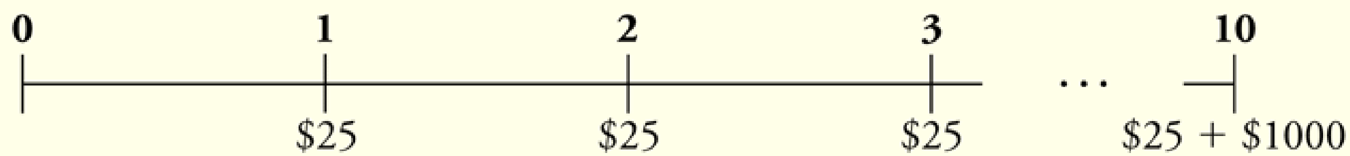
Problem

The U.S. Treasury has just issued a five-year, $1000 bond with a 5% coupon rate and semiannual coupons. What cash flows will you receive if you hold this bond until maturity?

Textbook Example 8.2 (cont'd)

Solution

The face value of this bond is \$1000. Because this bond pays coupons semiannually, from Eq. 8.1, you will receive a coupon payment every six months of CPN = \$1000 × 5%/2 = \$25. Here is the timeline, based on a six-month period:



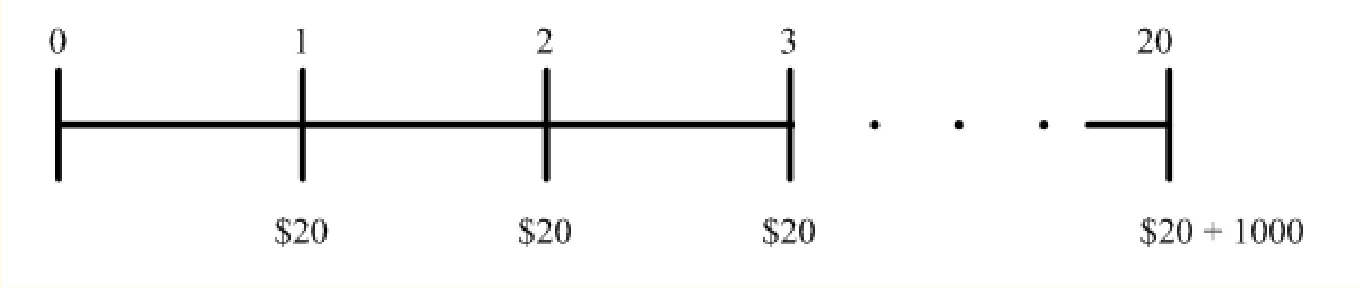
Note that the last payment occurs five years (10 six-month periods) from now and is composed of both a coupon payment of \$25 and the face value payment of \$1000.

Alternative Example 8.2

The U.S. Treasury has just issued a ten-year, $1000 bond with a 4% coupon and semi-annual coupon payments. What cash flows will you receive if you hold the bond until maturity?

Alternative Example 8.2 (cont'd)

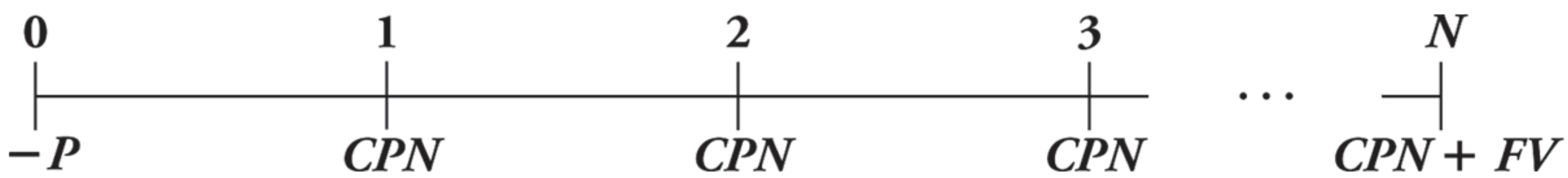
The face value of this bond is \$1000. Because this bond pays coupons semiannually, from Eq. 8.1 you will receive a coupon payment every six months of CPN = \$1000 X 4%/2 = \$20. Here is the timeline, based on a six-month period:



Note that the last payment occurs ten years (twenty six-month periods) from now and is composed of both a coupon payment of \$20 and the face value payment of \$1000.

Coupon Bonds (cont'd)

- Yield to Maturity
  - The YTM is the *single* discount rate that equates the present value of the bond's remaining cash flows to its current price.



  - Yield to Maturity of a Coupon Bond

$$P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}$$

Textbook Example 8.3

Computing the Yield to Maturity of a Coupon Bond

Problem

Consider the five-year, $1000 bond with a 5% coupon rate and semiannual coupons described in Example 8.2. If this bond is currently trading for a price of $957.35, what is the bond's yield to maturity?

Textbook Example 8.3 (cont'd)

Solution

Because the bond has 10 remaining coupon payments, we compute its yield y by solving:

$$957.35 = 25 \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^{10}}\right) + \frac{1000}{(1+y)^{10}}$$

We can solve it by trial-and-error or by using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	10		–957.35	25	1,000	
Solve for PV		**3.00%**				=RATE(10,25,–957.35,1000)

Therefore, $y = 3\%$. Because the bond pays coupons semiannually, this yield is for a six-month period. We convert it to an APR by multiplying by the number of coupon payments per year. Thus the bond has a yield to maturity equal to a 6% APR with semiannual compounding.

Financial Calculator Solution

- Since the bond pays interest semi-annually, the calculator should be set to 2 periods per year.

2ND	I/Y	2	ENTER

10	N
-957.35	PV
25	PMT
1,000	FV
CPT	I/Y

6

Alternative Example 8.3

- **Problem**
  - Consider the following semi-annual bond:
    - $1000 par value
    - 7 years until maturity
    - 9% coupon rate
    - Price is $1,080.55
  - **What is the bond’s yield to maturity?**

Alternative Example 8.3

- **Solution**

N = 7 years × 2 = 14

PMT = (9% × $1000) ÷ 2 = $45

2ND	I/Y	2	ENTER

14	N	
-1,080.55	PV	
45	PMT	
1,000	FV	
CPT	I/Y	7.5

Textbook Example 8.4

Computing a Bond Price from Its Yield to Maturity

Problem

Consider again the five-year, $1000 bond with a 5% coupon rate and semiannual coupons presented in Example 8.3. Suppose you are told that its yield to maturity has increased to 6.30% (expressed as an APR with semiannual compounding). What price is the bond trading for now?

Textbook Example 8.4 (cont'd)

Solution

Given the yield, we can compute the price using Eq. 8.5. First, note that a 6.30% APR is equivalent to a semiannual rate of 3.15%. Therefore, the bond price is

$$P = 25 \times \frac{1}{0.0315}\left(1 - \frac{1}{1.0315^{10}}\right) + \frac{1000}{1.0315^{10}} = \$944.98$$

We can also use the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	10	3.15%		25	1,000	
Solve for PV			**−944.98**			=PV(0.0315,10,25,1000)

Financial Calculator Solution

- Since the bond pays interest semi-annually, the calculator should be set to 2 periods per year.

2ND	I/Y	2	ENTER

10	N	
6.3	I/YR	
25	PMT	
1,000	FV	
CPT	PV	-944.98

Alternative Example 8.4

- **Problem**
  - Consider the bond in the Alternative Example 8.3.
    - Suppose its yield to maturity has increased to 10%
  - **What is the bond's new price**?

Alternative Example 8.4

- **Solution**

N = 7 years × 2 = 14

PMT = (9% × $1000) ÷ 2 = $45

2ND	I/Y	2	ENTER

14	N	
10	I/YR	
45	PMT	
1,000	FV	
CPT	PV	-950.51

8.2 Dynamic Behavior of Bond Prices

- Discount
  - A bond is selling at a **discount** if the price is less than the face value.
- Par
  - A bond is selling at **par** if the price is equal to the face value.
- Premium
  - A bond is selling at a **premium** if the price is greater than the face value.

Discounts and Premiums

- If a coupon bond trades at a discount, an investor will earn a return both from receiving the coupons and from receiving a face value that exceeds the price paid for the bond.
  - If a bond trades at a discount, its yield to maturity will exceed its coupon rate.

Discounts and Premiums (cont'd)

- If a coupon bond trades at a premium it will earn a return from receiving the coupons but this return will be diminished by receiving a face value less than the price paid for the bond.
- Most coupon bonds have a coupon rate so that the bonds will *initially* trade at, or very close to, par.

Discounts and Premiums (cont'd)

Table 8.1 Bond Prices Immediately After a Coupon Payment

When the bond price is …	greater than the face value	equal to the face value	less than the face value
We say the bond trades	"above par" or "at a premium"	"at par"	"below par" or "at a discount"
This occurs when	Coupon Rate > Yield to Maturity	Coupon Rate = Yield to Maturity	Coupon Rate < Yield to Maturity

Textbook Example 8.5

Determining the Discount or Premium of a Coupon Bond

Problem

Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity of each bond is 5%, what is the price of each bond per $100 face value? Which bond trades at a premium, which trades at a discount, and which trades at par?

Textbook Example 8.5 (cont'd)

Solution

We can compute the price of each bond using Eq. 8.5. Therefore, the bond prices are

$$P(10\%\text{ coupon}) = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86 \qquad \text{(trades at a premium)}$$

$$P(5\%\text{ coupon}) = 5 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$100.00 \qquad \text{(trades at par)}$$

$$P(3\%\text{ coupon}) = 3 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$69.26 \qquad \text{(trades at a discount)}$$

Financial Calculator Solution

2ND I/Y 1 ENTER

30 N

5 I/YR

10 PMT

100 FV

CPT PV -176.86

Financial Calculator Solution (cont'd)

2ND	I/Y	1	ENTER

30	N	
5	I/YR	
5	PMT	
100	FV	
CPT	PV	-100

Financial Calculator Solution (cont'd)

2ND	I/Y	1	ENTER

30	N	
5	I/YR	
3	PMT	
100	FV	
CPT	PV	-69.26

Time and Bond Prices

- Holding all other things constant, a bond's yield to maturity will not change over time.
- Holding all other things constant, the price of discount or premium bond will move towards par value over time.
- If a bond's yield to maturity has not changed, then the IRR of an investment in the bond equals its yield to maturity even if you sell the bond early.

Textbook Example 8.6

The Effect of Time on the Price of a Coupon Bond

Problem

Consider a 30-year bond with a 10% coupon rate (annual payments) and a $100 face value. What is the initial price of this bond if it has a 5% yield to maturity? If the yield to maturity is unchanged, what will the price be immediately before and after the first coupon is paid?

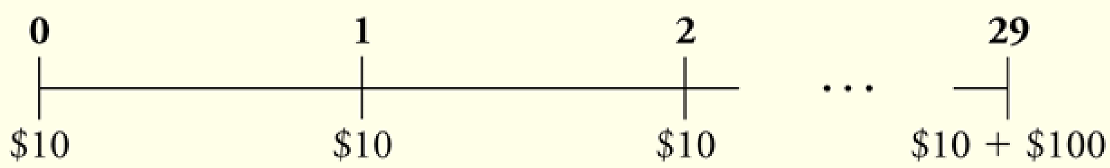
Textbook Example 8.6 (cont'd)

Solution

We computed the price of this bond with 30 years to maturity in Example 8.5:

$$P = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$$

Now consider the cash flows of this bond in one year, immediately before the first coupon is paid. The bond now has 29 years until it matures, and the timeline is as follows:



Again, we compute the price by discounting the cash flows by the yield to maturity. Note that there is a cash flow of $10 at date zero, the coupon that is about to be paid. In this case, it is easiest to treat the first coupon separately and value the remaining cash flows as in Eq. 8.5:

$$P(\text{just before first coupon}) = 10 + 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$185.71$$

Textbook Example 8.6 (cont'd)

Note that the bond price is higher than it was initially. It will make the same total number of coupon payments, but an investor does not need to wait as long to receive the first one. We could also compute the price by noting that because the yield to maturity remains at 5% for the bond, investors in the bond should earn a return of 5% over the year: $176.86 × 1.05 = $185.71.

What happens to the price of the bond just after the first coupon is paid? The timeline is the same as that given earlier, except the new owner of the bond will not receive the coupon at date zero. Thus, just after the coupon is paid, the price of the bond (given the same yield to maturity) will be

$$P(\text{just after first coupon}) = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$175.71$$

The price of the bond will drop by the amount of the coupon ($10) immediately after the coupon is paid, reflecting the fact that the owner will no longer receive the coupon. In this case, the price is lower than the initial price of the bond. Because there are fewer coupon payments remaining, the premium investors will pay for the bond declines. Still, an investor who buys the bond initially, receives the first coupon, and then sells it, earns a 5% return if the bond's yield does not change: (10 + 175.71)/176.86 = 1.05.

Financial Calculator Solution

- Initial Price

	30	N	
	5	I/YR	
	10	PMT	
	100	FV	
CPT		PV	-176.86

Financial Calculator Solution (cont'd)

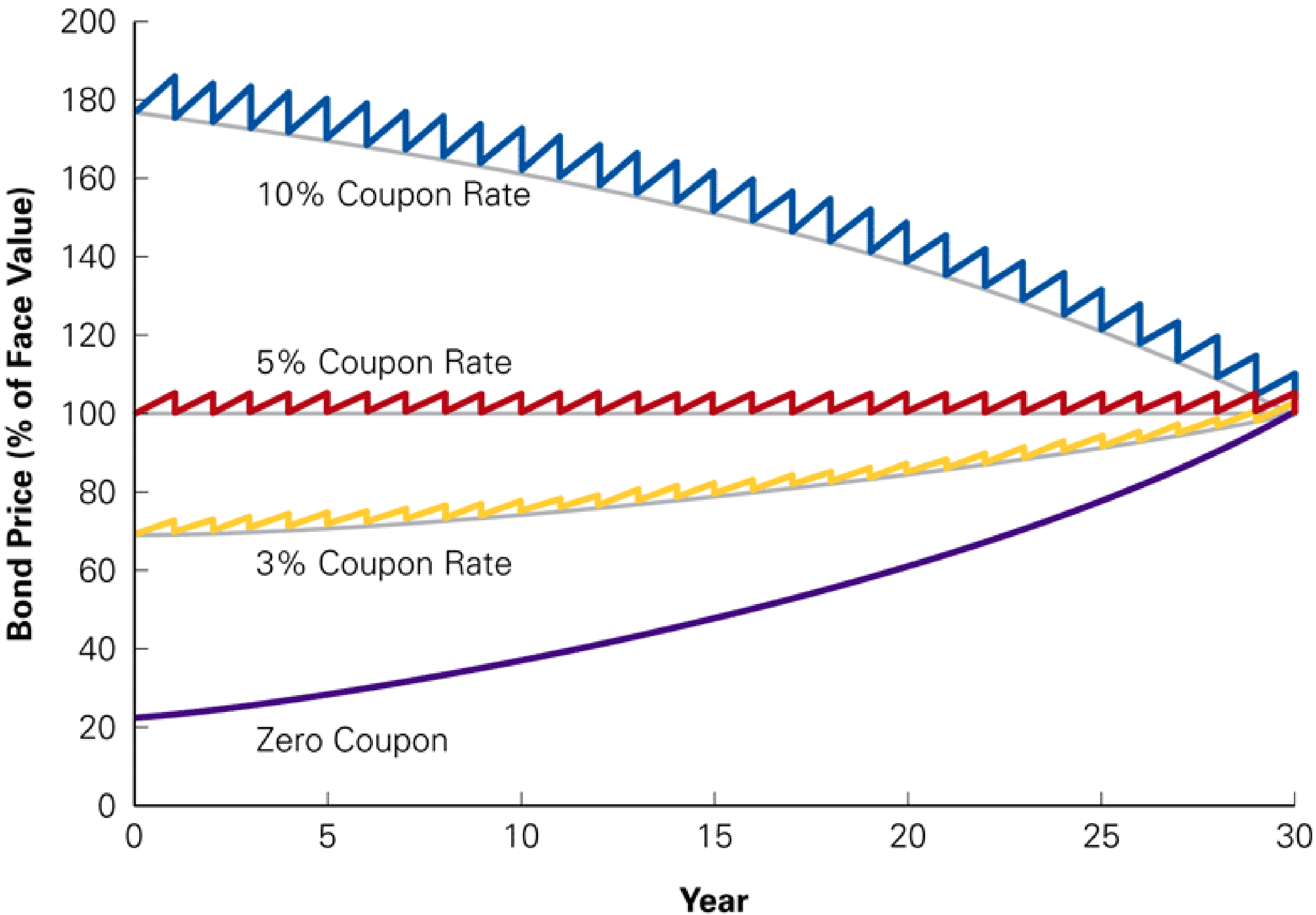
- Price just after first coupon

29	N
5	I/YR
10	PMT
100	FV
CPT	PV

-175.71

- Price just before first coupon
  - $175.71 + $10 = $185.71

Figure 8.1 The Effect of Time on Bond Prices



Interest Rate Changes and Bond Prices

- There is an inverse relationship between interest rates and bond prices.
  - As interest rates and bond yields rise, bond prices fall.
  - As interest rates and bond yields fall, bond prices rise.

Interest Rate Changes and Bond Prices (cont'd)

- The sensitivity of a bond’s price to changes in interest rates is measured by the bond’s **duration**.
  - Bonds with high durations are highly sensitive to interest rate changes.
  - Bonds with low durations are less sensitive to interest rate changes.

Textbook Example 8.7

The Interest Rate Sensitivity of Bonds

Problem

Consider a 15-year zero-coupon bond and a 30-year coupon bond with 10% annual coupons. By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%?

Textbook Example 8.7 (cont'd)

Solution

First, we compute the price of each bond for each yield to maturity:

Yield to Maturity	15-Year, Zero-Coupon Bond	30-Year, 10% Annual Coupon Bond
5%	$\frac{100}{1.05^{15}} = \$48.10$	$10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$
6%	$\frac{100}{1.06^{15}} = \$41.73$	$10 \times \frac{1}{0.06}\left(1 - \frac{1}{1.06^{30}}\right) + \frac{100}{1.06^{30}} = \$155.06$

The price of the 15-year zero-coupon bond changes by (41.73 − 48.10)/48.10 = −13.2% if its yield to maturity increases from 5% to 6%. For the 30-year bond with 10% annual coupons, the price change is (155.06 − 176.86)/176.86 = −12.3%. Even though the 30-year bond has a longer maturity, because of its high coupon rate its sensitivity to a change in yield is actually less than that of the 15-year zero coupon bond.

Figure 8.2 Yield to Maturity and Bond Price Fluctuations Over Time